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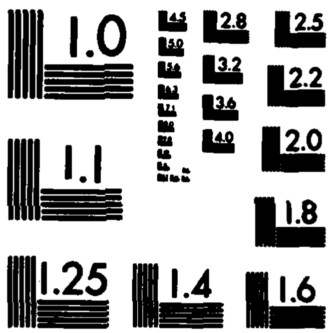

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A FOURTH ORDER DECONVOLUTION TECHNIQUE FOR NONGAUSSIAN LINEAR PROCESSES*

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Introduction. In Lii and Rosenblatt (1982) a deconvolution scheme for nonGaussian linear processes making use of third order moments (or spectra) was presented. This is appropriate for such processes with nonzero third order central moments. However, if the third order moments are zero (this could happen in the case of symmetric distributions) it is appropriate to look for a fourth order technique that would be effective. Such a scheme is presented and discussed in this paper together with some illustrative examples.

We give a brief sketch of the theoretical background. Let v_t, $t = \ldots,-1,0,1,\ldots$ be independent, identically distributed random variables with mean zero and variance one. Consider a sequence of real constants $\{\alpha_j\}$ with

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$$\sum_{j=-\infty}^{\infty} \alpha_j^2 < \infty .$$

Let $\{x_t\}$ be the linear process

$$x_t = \sum_{j=-\infty}^{\infty} \alpha_j v_{t-j} . \tag{1}$$

Introduce the z-transform $\alpha(z) = \sum_j \alpha_j z^j$ corresponding to the process $\{x_t\}$. We should like to estimate $\alpha(e^{-i\lambda})$ (λ real) from observations only on the process $\{x_t\}$ and use this estimate to deconvolve the process $\{x_t\}$ and estimate the ξ_t's. As noted in Rosenblatt (1980), in the Gaussian case one can only estimate the modulus of $\alpha(e^{-i\lambda})$, and it is only in the nonGaussian case that one can also estimate the argument of $\alpha(e^{-i\lambda})$. Of course, the spectral density of $\{x_t\}$ is

$$f(\lambda) = \frac{1}{2\pi} |\alpha(e^{-i\lambda})|^2 .$$

In some geophysical contexts a nonGaussian model like that discussed here has been proposed. A basic concern is that of deconvolution, estimating the a_j's and v_j's. A discussion of such questions with some of the geophysical background can be found in Donoho (1981), Godfrey and Rocca (1981), and Wiggins (1978).

The following lemma was proved in Lii and Rosenblatt (1982) with the type of argument suggested in Rosenblatt (1980).

<u>Lemma</u>. *Consider a nonGaussian linear process* $\{x_t\}$ (see (1)) *with the independent random variables having all their moments finite. Let*

$$\sum |j| |\alpha_j| < \infty$$

and assume $\alpha(e^{-i\lambda}) \neq 0$ *for all* λ. *The function* $\alpha(e^{-i\lambda})$ *can then be identified in terms of observations on only* $\{x_t\}$ *up to an*

indeterminate integer a *in a factor* $e^{ia\lambda}$ *and an indeterminate sign of* $\alpha(1) = \sum \alpha_k$. *For this result it is sufficient to have some finite moment of order* $k > 2$ *with cumulant* $\gamma_k \neq 0$.

The k^{th} order cumulant spectral density of the process $\{x_t\}$ can be seen to be

$$b_k(\lambda_1,\ldots,\lambda_{k-1}) = \frac{\gamma_k}{(2\pi)^{k-1}} \alpha(e^{-i\lambda_1}) \ldots (e^{-i\lambda_{k-1}}) \alpha(e^{i(\lambda_1 + \cdots + \lambda_{k-1})}) \ .$$

If one sets

$$h(\lambda) = \arg \left\{ \alpha(e^{-i\lambda}) \frac{\alpha(1)}{|\alpha(1)|} \right\}$$

then it can be shown that

$$h(\lambda_1) + \cdots + h(\lambda_{k-1}) - h(\lambda_1 + \cdots + \lambda_{k-1})$$
$$= \arg \left[\left\{ \frac{\alpha(1)}{|\alpha(1)|} \right\}^k \gamma_k^{-1} \, b_k(\lambda_1,\ldots,\lambda_{k-1}) \right]$$

The case in which $k = 4$ is of particular interest to us. From this point on we will often delete the subscript $k = 4$ but understand that we are dealing with 4^{th} order cumulant spectral estimates. The relation

(2) $$h(\lambda) = \int_0^\lambda \{h'(u) - h'(0)\} du + c\lambda = h_1(\lambda) + c\lambda \ .$$

Since $h(\pi)$ must be an integral multiple of π (because the α_j's are real) we can rewrite (2) as

$$h^*(\lambda) = h_1(\lambda) - \frac{h_1(\pi)}{\pi}\lambda + a\lambda$$

with a an indeterminate integer. Further

$$h'(0) - h'(\lambda) = \lim_{\Delta\to 0}\frac{1}{2\Delta}\{h(\lambda) + 2h(\Delta) - h(\lambda+2\Delta)\}$$

and

$$h(\lambda) + 2h(\Delta) - h(\lambda+2\Delta) = \arg\{b(\lambda,\Delta,\Delta)\}$$

up to a sign. We shall consider the question of estimating $h_1(\lambda)$. Set $\Delta = \Delta(n)$, $(2k+1)\Delta = \lambda$ and consider $\Delta = \Delta(n) \to 0$ as $n \to \infty$. Clearly $b(0,0,0)$ is positive. Notice that

$$\begin{aligned} h_1(\lambda) &= h(\lambda) - h'(0)\lambda \\ &\cong h((2k+1)\Delta) - \frac{h(\Delta)}{\Delta}(2k+1)\Delta \\ &= \sum_{j=0}^{k-1}\{h((2j+1)\Delta) + 2h(\Delta) - h((2j+3)\Delta)\} \\ &= -\sum_{j=0}^{k-1}\arg b((2j+1)\Delta,\Delta,\Delta) \ . \end{aligned}$$

This suggests taking

$$H_n(\lambda) = -\sum_{j=0}^{k-1} \arg {}_n b((2j+1)\Delta,\Delta,\Delta) \tag{3}$$

as a possible estimate of $h_1(\lambda)$. We shall assume that ${}_n b$ is a consistent sequence of estimates of the fourth order cumulant spectral density. Conditions for existence of such a sequence of estimators can be found in Brillinger and Rosenblatt (1967). Here n denotes the sample size. Take

$$\theta_n(\lambda,\mu,\eta) = \arctan(\mathrm{Im}\ {}_n b(\lambda,\mu,\eta)/\mathrm{Re}\ {}_n b(\lambda,\mu,\eta))$$

as an estimate of

$$\theta(\lambda,\mu,\eta) = \arg b(\lambda,\mu,\eta) .$$

Then just as in Lii and Rosenblatt (1982) one has

$$\begin{aligned}
&\theta_n(\lambda,\mu,\eta) - \theta(\lambda,\mu,\eta) \\
&\quad = -\frac{\mathrm{Im}\ b(\lambda,\mu,\eta)}{|b(\lambda,\mu,\eta)|^2}\{\mathrm{Re}\ {}_n b(\lambda,\mu,\eta) - \mathrm{Re}\ b(\lambda,\mu,\eta)\} \\
&\quad + \frac{\mathrm{Re}\ b(\lambda,\mu,\eta)}{|b(\lambda,\mu,\eta)|^2}\{\mathrm{Im}\ {}_n b(\lambda,\mu,\eta) - \mathrm{Im}\ b(\lambda,\mu,\eta)\} \\
&\quad + o_p({}_n b(\lambda,\mu,\eta) - b(\lambda,\mu,\eta)) .
\end{aligned}$$

Suppose $H_n(\lambda)$ is taken as an estimate of $h_1(\lambda)$. If $\alpha(e^{-i\lambda}) \in C^2$, the weight function of ${}_n b$ is symmetric and bandlimited with bandwidth Δ, $\Delta(n) \to 0$, $\Delta^3 n \to \infty$ as $n \to \infty$, then

$$H_n(\lambda) - h_1(\lambda) = R_n(\lambda) + o_p(H_n(\lambda) - h_1(\lambda))$$

where

$$R_n(\lambda) = \sum \left[\frac{\operatorname{Im} b((2j+1)\Delta,\Delta,\Delta)}{|b((2j+1)\Delta,\Delta,\Delta)|^2} \{\operatorname{Re} {}_n b(2j+1)\Delta,\Delta,\Delta) - \operatorname{Re} b((2j+1)\Delta,\Delta,\Delta)\} \right.$$

$$(4) \qquad \left. - \frac{\operatorname{Re} b((2j+1)\Delta,\Delta,\Delta)}{|b((2j+1)\Delta,\Delta,\Delta)|^2} \{\operatorname{Im} {}_n b((2j+1)\Delta,\Delta,\Delta) - \operatorname{Im} b((2j+1)\Delta,\Delta,\Delta)\} \right] .$$

One can show that

$$E\, R_n(\delta) \sim -\int_0^\lambda \frac{1}{2} \{b(u_1,0,0)\}^{-1} \sum_{j,k} A_{jk} D_{u_j} D_{u_k} (2-\delta_{jk})$$

$$b(u_1,u_2,u_3)\Big|_{u_2=u_3=0} \, du\, \Delta$$

$$+ o(\Delta)$$

where the A_{jk} are the moments

$$A_{jk} = \int u_k u_k \, W(u_1,u_2,u_3) du_1 du_2 du_3$$

and D_{u_j} is the partial derivative with respect to u_j. Further

$$\operatorname{cov}(R_n(\lambda),R_n(\mu)) \cong \pi \frac{f^2(0)}{\Delta^4 n} \int_0^{\min(\lambda,\mu)} \{f^2(u)/|b(u,0,0)|^2\} du$$

$$\int W^2(u,v,w) du dv dw \, .$$

$$= \frac{4\pi^3}{\Delta^4 n\gamma^2} \min(\lambda,\mu) \int W^2(u,v,w) du dv dw$$

Here W is the standardized weight function of the fourth order cumulant spectral density estimate.

Computational methods. We consider computational schemes for computing $H_n(\lambda)$ in (3) as an estimate of $h_1(\lambda)$. Given a sample $\{x_t\}$ of size $n = mN$ break up the sample into m disjoint subsections of equal length N so that the variance of the trispectral estimate (estimate of the cumulant spectral density of fourth order) of each section is not too large. This is particularly important in the trispectral case since the variance of the fourth order periodogram used in constructing trispectral estimates is proportional to N^2. Then choose a grid of points $\lambda_j = (2j+1)\Delta$ in $(0,2\pi)$, $j = 0,1,\ldots,M$, $\Delta = 2\pi L/N$ for a suitable integer L. Form the trispectral estimates ${}_sb(\lambda_j,\Delta,\Delta)$, $s = 1,\ldots,m$, for each of the m sections of length N by using a weight function of bandwidth $2\Delta,\Delta,\Delta$ in each component since $\lambda_j - \lambda_{j-1} = 2\Delta$. Average the estimates obtained from each of the subsections of length N at λ_j to obtain the final estimate ${}_n\hat{b}(\lambda_j,\Delta,\Delta)$. Compute $\theta_n(\lambda_j) =$ $\arg\{{}_n\hat{b}(\lambda_j,\Delta,\Delta)\}$ and form

$$H_n(\lambda_\ell) = -\sum_{j=0}^{\ell-1} \theta_n(\lambda_j), \quad \ell = 1,2,\ldots,M+1 .$$

We set $H_n(0) = 0$ since $h(0) = 0$ and estimate $H_n(\lambda_0) = H_n(\Delta)$ by an interpolation between 0 and $H_n(\lambda_1) = H_n(3\Delta)$. Then coefficient α_k in the trigonometric expansion of $\alpha(e^{-i\lambda})$ can be estimated by

$$(5)\quad \hat{a}_k = \frac{1}{2\pi}\int_0^{2\pi} \hat{a}(e^{-i\lambda})e^{ik\lambda}\,d\lambda$$

$$\cong \frac{1}{M+2}\sum_{j=0}^{M+1} (2\pi\ f_n(\lambda_j))^{\frac{1}{2}} \exp\{i(H_n(\lambda_j) - \frac{H_n(\pi)}{\pi}\lambda_j) + ik\lambda_j\}$$

where $f_n(\lambda_j)$ is a consistent estimate of the spectral density $f(\lambda)$ of $\{x_t\}$. The spectral density estimate $f_n(\lambda_j)$ is formed as follows: Form smoothed periodograms with bandwidth $\Delta_1 \leq 2\Delta$ from each of the m subsections of length N and average the m smoothed periodograms to get $f_n(\lambda_j)$. $f_n(o)$ is estimated by extrapolation. Presumably one could improve formula (5) by using a more refined approximation to the integral based on the trapezoidal rule or Simpson's rule. Also an extrapolation procedure could be used at the end points since $0,\Delta,3\Delta,\ldots$ are not equally spaced.

We now describe an alterative procedure for estimating $h_1(\lambda)$. Note that

$$\sum_{j=1}^{k-1} \arg b(j\Delta,\Delta,\Delta)$$

$$= \sum_{j=1}^{k-1} \{h(j\Delta) + 2h(\Delta) - h(j\Delta+2\Delta)\}$$

$$= 2[kh(\Delta) - h(k\Delta)] + B$$

with

$$B = h(2\Delta) - h(\Delta) + h(k\Delta) - h((k+1)\Delta)\ .$$

Thus if $\lambda = k\Delta$

(6) $$h_1(\lambda) = h(\lambda) - h'(0)\lambda$$

$$\cong -\frac{1}{2}\sum_{j=1}^{k-1} \arg b(j\Delta,\Delta,\Delta) - \frac{1}{2} B .$$

If Δ is small we would expect B to be small also. This suggests that a plausible estimate of $h_1(\lambda)$ could be given by

$$G_n(\lambda) = -\frac{1}{2}\sum_{j=1}^{k-1} \arg {}_n b(j\Delta,\Delta,\Delta) .$$

The estimate $G_n(\lambda)$ may have an additional bias relative to the estimate $H_n(\lambda)$ because of the term $-\frac{1}{2} B$ in (6). However, a full comparison of the two estimates is difficult to make. There are advantages and disadvantages to each. The estimate actually used in the computational illustrations discussed later is $G_n(\lambda)$.

To deconvolve the observed signal $\{x_t\}$ and obtain estimates of $\{v_t\}$, we form

$$\hat{v}_t = \hat{\alpha}^{-1}(L)x_t$$

where L is the backward shift operator. When $\alpha(e^{-i\lambda})$ is one sided polynomial of order q (this corresponds to $\{x_t\}$ a moving average of order q) methods using a partial fraction expansion of $\hat{\alpha}^{-1}(L)$ by computing the roots of $\hat{\alpha}(z)$ are described in Lii and Rosenblatt (1982).

To avoid finding an appropriate finite parameter model for $\{x_t\}$ and dealing with the sensitivity of root location in terms of their dependence on coefficients, we note that one can find the deconvolution weights by inverting $\hat{\alpha}(e^{-i\lambda})$ directly. Let $b(e^{-i\lambda}) = \hat{\alpha}(e^{-i\lambda})$. Then the coefficient b_k in the expansion

$$b(e^{-i\lambda}) = \sum b_k e^{-ik\lambda}$$

can be computed by using

$$b_k = \frac{1}{2\pi}\int_0^{2\pi} [2\pi\ f_n(\lambda)]^{-\frac{1}{2}} \exp\{i(-G_n(\lambda) + \frac{G_n(\pi)}{\pi}\lambda + k\lambda)\} \tag{7}$$

$$\cong \frac{1}{J+1}\sum_{j=1}^{J} [2\pi\ f_n(\lambda_j)]^{-\frac{1}{2}} \exp\{i(-G_n(\lambda_j) + \frac{G_n(\pi)}{\pi}\lambda_j + k\lambda_j)\}\ ,$$

$k = \ldots,-1,0,1,\ldots$. Usually we find uitable integers k_1 and k_2 and use the real part of b_k for $k = k_1,\ldots,k_2$ as deconvolution weights since we are dealing with a real process. In the examples discussed below the choice was $k_1 = -9$ and $k_2 = 9$.

Examples. A few simple examples are presented here to illustrate the computational procedures. The model considered is

$$x_t = v_t + {}_1v_{t-1} + {}_2v_{t-2} \quad t = 1,\ldots,640$$

where

$$v_t = (v'_t - \bar{v}'_t)/s$$

$$\bar{v}'_t = \sum_{t=1}^{640} v'_t/640$$

$$s^2 = \sum_{t=1}^{640} (v'_t - \bar{v}'_t)^2/640$$

and the v_t's are independent and identically distributed. The general computational set up is the same as that in Lii and Rosenblatt (1982). All the examples deal with schemes generated with coefficients (and roots) as specified in Table 1.

Table 1. Coefficients and roots for four cases

Case	Coefficients			Roots	
	α_0	α_1	α_2	r_1	r_2
1	1.0	- .833	0.167	2.0	3.0
2	1.0	-2.333	0.667	0.5	3.0
3	1.0	-3.50	1.50	2.0	.333
4	1.0	-5.0	6.0	0.5	.333

In the first set of examples (four) v'_t is the exponential distribution with parameter 1 generated by GGEXN in IMSL. Although the third order cumulant of v'_t is nonzero, the fourth order technique considered in this paper can be used. Table 2 compares the estimated coefficients in each of the cases as computed by third and fourth order techniques.

Table 2.

Case	Third order $\hat{\alpha}_0$	Third order $\hat{\alpha}_1$	Third order $\hat{\alpha}_2$	Fourth order $\hat{\alpha}_0$	Fourth order $\hat{\alpha}_1$	Fourth order $\hat{\alpha}_2$
1	1.05	-0.661	.05	.6955	- .747	.0101
2	.8358	-2.132	.796	1.011	-2.043	.644
3	1.34	-3.32	1.17	1.44	-3.25	1.18
4	1.05	-3.23	6.56	.805	-4.456	5.925

The deconvolution of case 2 using the third order method is shown in Figure 1a. This can be compared with deconvolution by the fourth order method which is given in Figure 1b. Both deconvolutions in Figure 1 involved computation of roots. The mean square errors of $v_t - \hat{v}_t$ for the third order and fourth order methods were .045 and .094 respectively.

In the second set of examples, the v_t' distribution was the symmetric double exponential. The estimated coefficients for the four cases, using a fourth order method, are given in Table 3.

Table 3

Case	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$
1	1.08	- .3886	- .1043
2	.8835	-2.121	.8028
3	1.874	-2.544	1.153
4	1.805	-3.22	3.865

The deconvolution for case 2 using location of roots is given in Figure 2. The mean square error of $v_t - \hat{v}_t$ is .01559 while the variance of v_t is .25.

In the last set of examples v_t' has a symmetric Pareto distribution. First uniform random numbers U_i (on the interval (0,1)) are generated by GGUW in IMSL. Then the transformation $y_i = (U_i)^{-1/5}$ is used to obtain random numbers having a Pareto distribution with density $f(y) = 5y^{-6}$, $y \geq 1$. The v_t''s are obtained by randomly changing the sign of $y_i - 1$ with probability .5. The estimated coefficients in the four cases using a fourth order procedure are given in Table 4.

Table 4

Case	$\hat{\alpha}_0$	$\hat{\alpha}_1$	$\hat{\alpha}_2$
1	1.040	-.6566	.1919
2	1.037	-2.095	.5926
3	1.265	-3.32	1.252
4	1.599	-5.158	4.799

Figure 3a gives the result of deconvolution of case 2 using computation of roots. Figure 3b gives the result of direct deconvolution in case 2. The mean square error of $v_t - \hat{v}_t$ is .0059 and .0011 for the first and second deconvolution procedures respectively.

Comments on computation. A decision as to when to use a third or fourth order deconvolution procedure could be based on estimates of third and fourth order cumulants. A larger estimate (in absolute magnitude) for a specific cumulant would suggest that one could with some condifence prefer using the deconvolution procedure of the same order. Of course, if the cumulants were too small in magnitude there wouldn't be much point in attempting the deconvolution.

The sample size used in the illustrative computations is 640. In the ordinary usage this would be thought of as a large sample. One thing that is apparent is the relative effectiveness of the deconvolution procedure independent of the tail behavior of the v_t distribution. But one can say more. In a certain sense the sample size 640 is moderate (perhaps even small). Suppose we look at the question of estimating the third and fourth central moments when one has a sample size of n observations. The first order expressions for the variances of the standard estimates of third and fourth central moments are

$$\frac{\mu_6-6\mu_2\mu_4-\mu_3^2+9\mu_2^3}{n} \tag{8}$$

and

$$\frac{\mu_8-8\mu_3\mu_5-\mu_4^2+16\mu_2\mu_3^2}{n} \tag{9}$$

respectively (see Cramèr (1964)). Here μ_k is the k^{th} moment of the distribution in question. Suppose we look at the coefficient of $1/n$ in (8) for the case of an exponential distribution with density e^{-x} for $x > 0$. It is 195 and in terms of this the implication is that one would need a sample size of about 600 to get a variance of the order of magnitude of one. The case is much more extreme for the coefficient of $1/n$ in (9) for the case of a symmetric exponential density $e^{-|x|}\frac{1}{2}$. The coefficient is 39,744.

<u>Deconvolution weights</u>. Here we will sketch an argument that allows us to get an asymptotic approximation for the covariances of the principal random part of deconvolution weight estimates b_k. A similar argument can be used to obtain such an approximation for the covariances of the principal random part of the estimates $\hat{\alpha}_k$. Expression (7) can be rewritten as

$$b_k = \frac{2}{J+1}\sum_{j=1}^{J/2} [2\pi\ f_n(\lambda_j)]^{-\frac{1}{2}} \cos\{-G_n(\lambda_j) + \frac{G_n(\pi)}{\pi}\lambda_j + k\lambda_j\}\ .$$

Now

$$(10)\quad (f_n(\lambda_j))^{-\frac{1}{2}} = (E\ f_n(\lambda_j))^{-\frac{1}{2}}\left(1 + \frac{f_n(\lambda_j)-Ef_n(\lambda_j)}{Ef_n(\lambda_j)}\right)^{-\frac{1}{2}}$$

$$= (E\ f_n(\lambda_j))^{-\frac{1}{2}}\left(1 - \frac{1}{2}\frac{f_n(\lambda_j)-Ef_n(\lambda_j)}{Ef_n(\lambda_j)} + o(f_n(\lambda_j) - Ef_n(\lambda_j))\right)\ .$$

Further

$$(11)\quad \cos\left\{- G_n(\lambda_j) + \frac{G_n(\pi)}{\lambda}\lambda_j + k\lambda_j\right\}$$

$$= \cos\left\{- EG_n(\lambda_j) + \frac{EG_n(\pi)}{\pi}\lambda_j + k\lambda_j + \left[- G_n(\lambda_j) + EG_n(\lambda_j) + \frac{G_n(\pi)-EG_n(\pi)}{\pi}\lambda_j\right]\right\}$$

$$= \cos\left\{- EG_n(\lambda_j) + \frac{EG_n(\pi)}{\pi}\lambda_j + k\lambda_j\right\}$$

$$- \sin\left\{- EG_n(\lambda_j) + \frac{EG_n(\pi)}{\pi}\lambda_j + k\lambda_j\right\}$$

$$\left[- G_n(\lambda_j) + EG_n(\lambda_j) + \frac{G_n(\pi)-EG_n(\pi)}{\pi}\lambda_j\right]$$

$$+ o_p\left[- G_n(\lambda_j) + EG_n(\lambda_j) + \frac{G_n(\pi)-EG_n(\pi)}{\pi}\lambda_j\right]\ .$$

First it should be noted that the second term on the right of (10) will be of smaller order than the second term on the right of (11). This implies that the principal random part of b_k (the deterministic mean is neglected here) can be approximated by

$$\frac{2}{J+1}\sum_{j=1}^{J/2}(2\pi\ f(\lambda_j))^{-\frac{1}{2}}\sin\ (-\ h_1(\lambda_j)+k\lambda_j)$$

$$\left[-\ G_n(\lambda_j)+EG_n(\lambda_j)+\frac{G_n(\pi)-EG_n(\pi)}{\pi}\lambda_j\right].$$

The principal part of

$$-\ G_n(\lambda)+EG_n(\lambda)+\frac{G_n(\pi)-EG_n(\pi)}{\pi}\lambda,\ 0<\lambda<\pi\ ,$$

asymptotically has the covariance (the argument is like that given for $R_n(\lambda)$ in (4))

$$\frac{4\pi^4}{\Delta^4 n\gamma^2}\left\{\min\left(\frac{\lambda}{\pi},\frac{\mu}{\pi}\right)-\frac{\lambda\mu}{\pi^2}\right\}\int W^2(u,v,w)\ dudvdw$$

if $\Delta^3 n\to\infty$, $\Delta(n)\to 0$. This implies that the covariance of principal random parts of b_j, b_k are (j,k fixed)

$$\frac{2\pi}{\Delta^4 n\gamma^2}\int_0^{\pi}\!\!\int(f(\lambda)f(\mu))^{-\frac{1}{2}}\sin\ (h_1(\lambda)-k\lambda)$$

$$\sin\ (h_1(\mu)-j\mu)\left\{\min\left(\frac{\lambda}{\pi},\frac{\mu}{\pi}\right)-\frac{\lambda\mu}{\pi^2}\right\}d\lambda d\mu$$

$$\int W^2(u,v,w)dudvdw\ .$$

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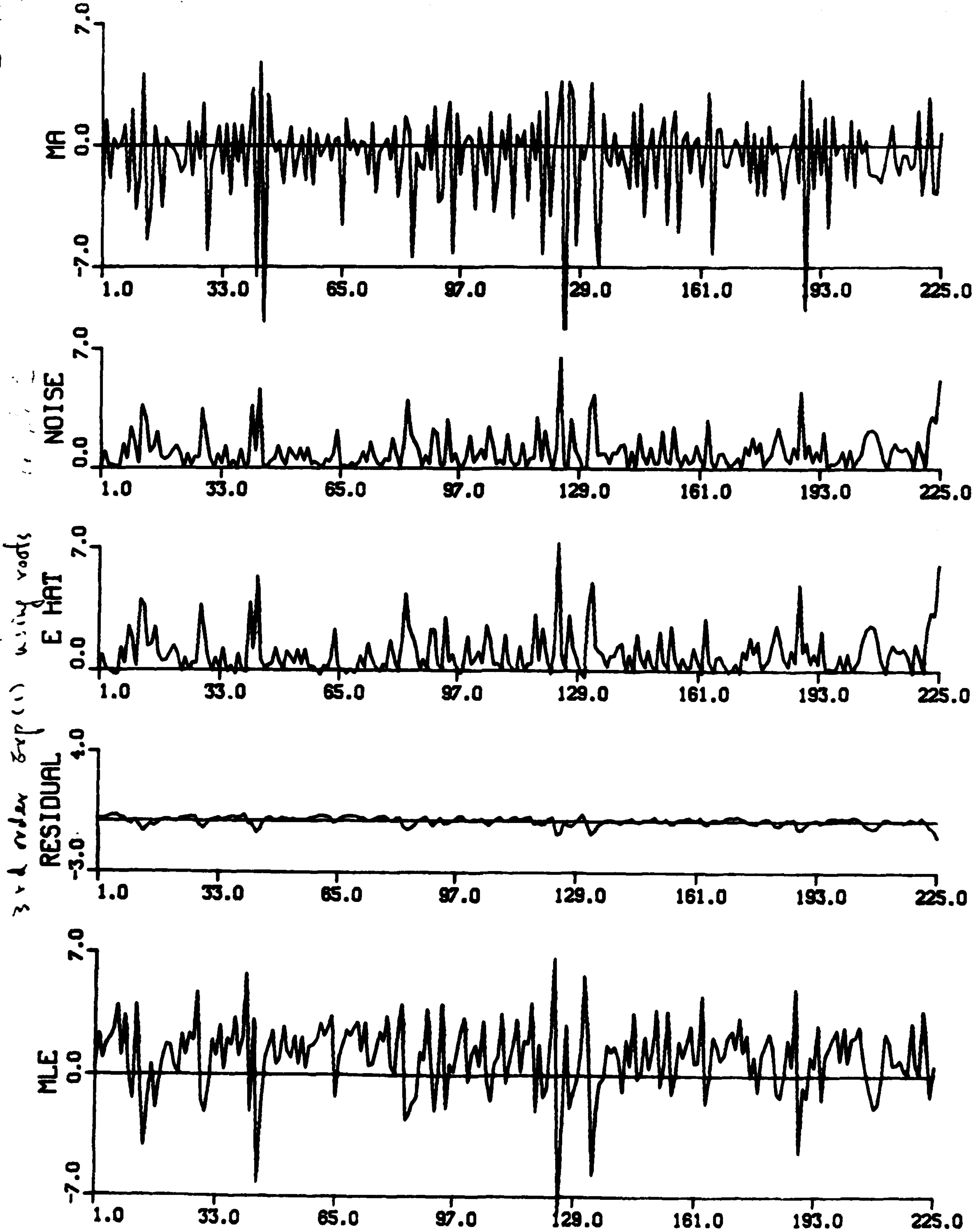
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3rd order exp(1) using roots

FIGURE 1a

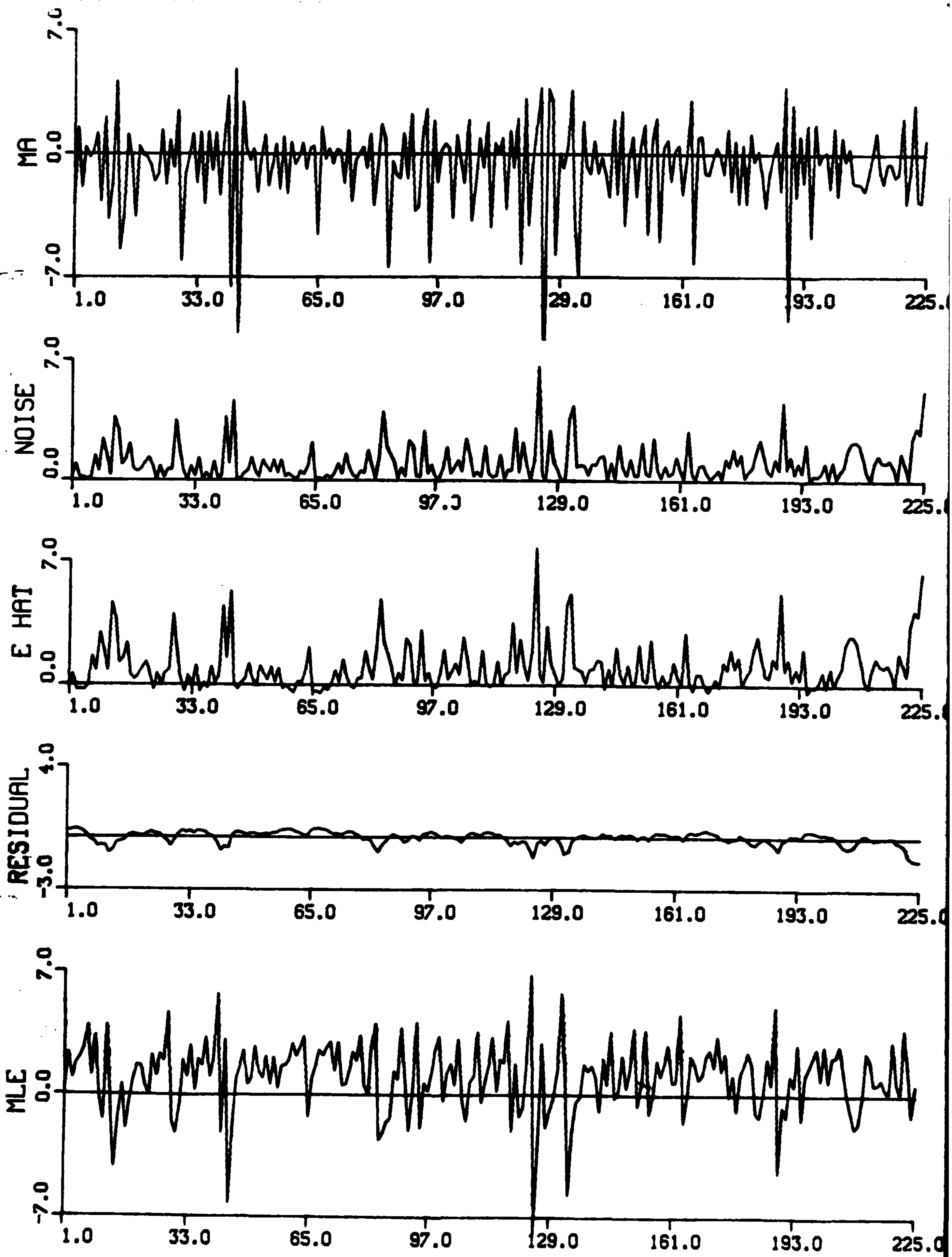
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FIGURE 1b

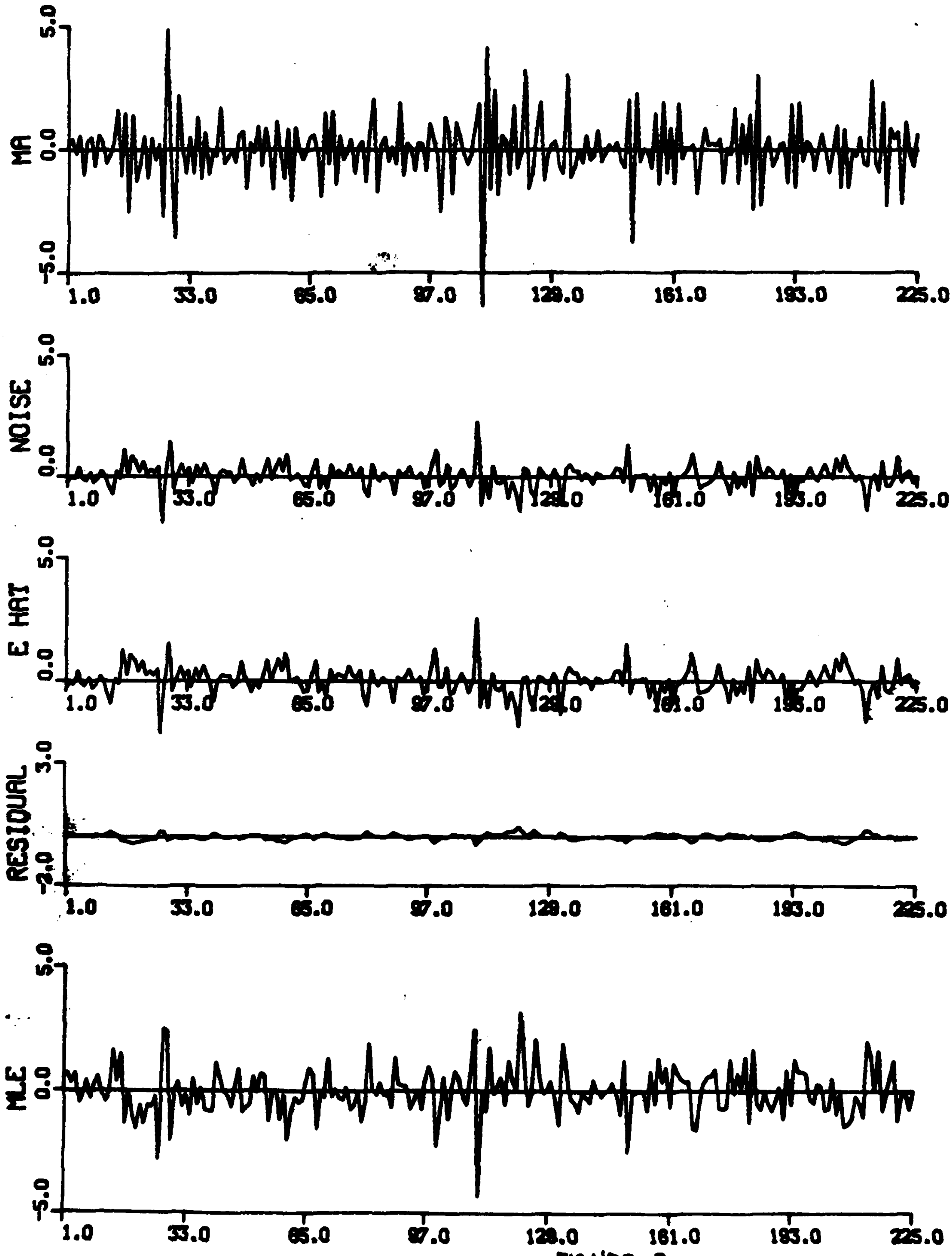
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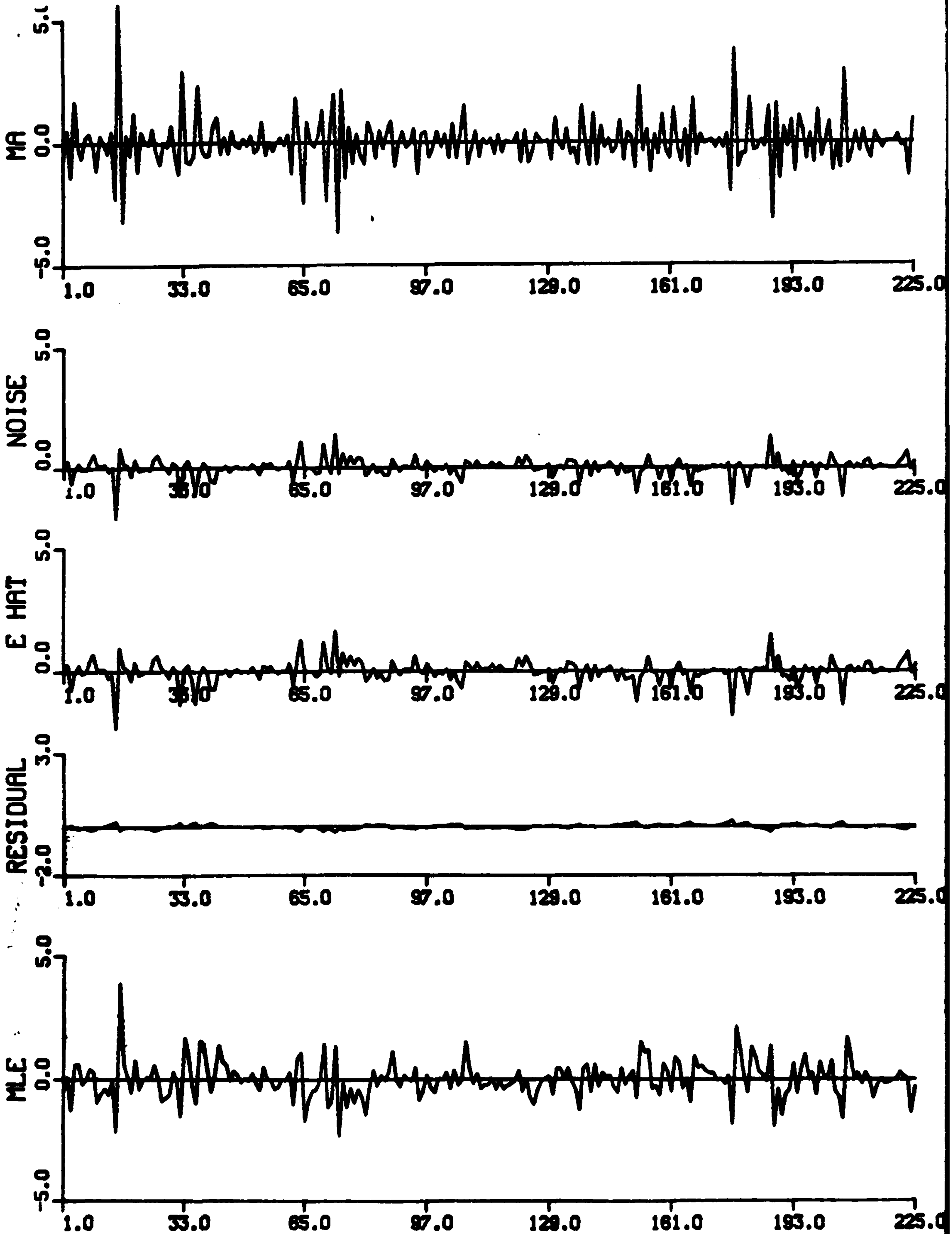
FIGURE 2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193.0
225.0

FIGURE 3a

Direct Deconvolution of Visible Pareto (c-s) ... 2

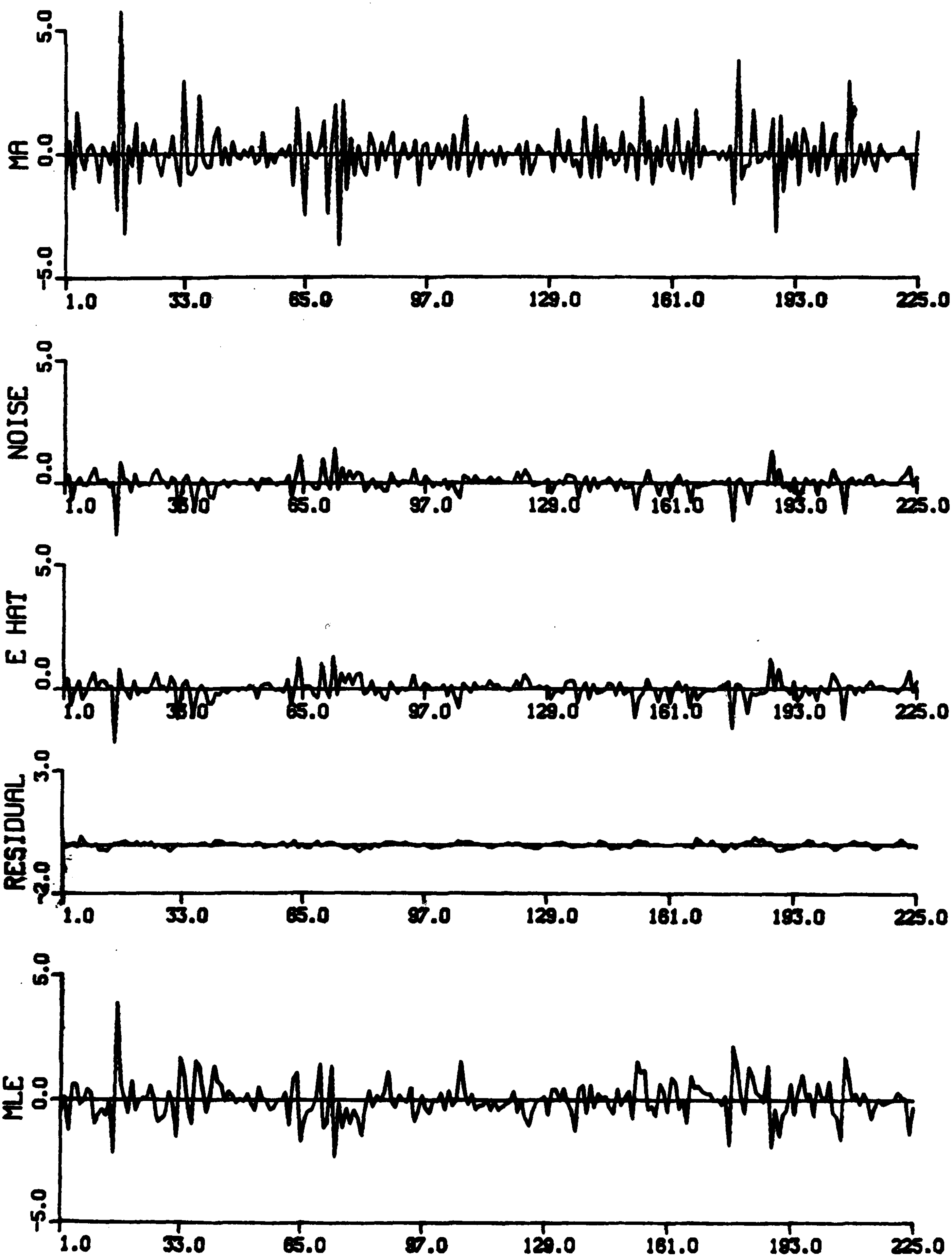

FIGURE 3b

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Deconvolution, nonGaussian, fourth order, cumulant spectrum, phase

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

A deconvolution procedure appropriate for nonGaussian linear processes with symmetric (or nonsymmetric distributions) is presented. The procedure makes use of estimates of the fourth order cumulant spectral density. Large sample properties of aspects of the deconvolution technique are described. Illustrative examples are given.

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